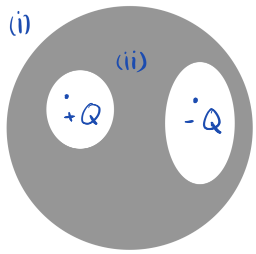
Electricity and Magnetism, Exam 2, 15/03/2019

7 questions, 30 points total

> Write your name and student number on the answer sheet. Use of a calculator is allowed. You may make use of the provided formula sheet. The same notation is used as in the book, i.e. a bold-face $\mathbf{A}$ is a vector, $\hat{\mathbf{x}}$ is the unit vector in the x-direction, and T is a scalar.

1. *3 points*

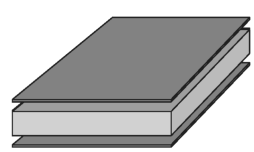


An uncharged solid conductor has two cavities carved out. In one of them we place a charge $-Q$, in the other one a charge $+Q$. The situation is illustrated on the left. What can we say about the electric field in the regions (i) and (ii)?

A. region (i) zero, region (ii) zero
B. region (i) nonzero, region (ii) zero
C. region (i) zero, region (ii) nonzero
D. region (i) nonzero, region (ii) nonzero

Answer: In both regions, the electric field is zero. Inside a conductor the electric field is always zero; outside, the total enclosed charge is zero, and the conductor shields all the electric field from the charges in the cavities.
Answer model: all or nothing

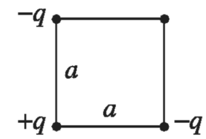
2. *5 points*
The space between two parallel metal plates is *half-filled* with an insulating material of dielectric constant ϵ_r, as illustrated in the figure on the right. For a charge density $+\sigma$ on the top plate and $-\sigma$ on the lower plate, and plate distance d, find the electric field in between the two plates (both inside the dielectric and the air-filled part) and from that the voltage difference between the two metal plates.



Answer: Problem 4.19a): The electric field can be found from $\mathbf{D} = \sigma$ *to be* $\mathbf{E} = \sigma/\epsilon_0$ *in air and* $\mathbf{E} = \sigma/\epsilon$ *in the dielectric. So* $V = \frac{\sigma}{\epsilon_0}\frac{d}{2} + \frac{\sigma}{\epsilon}\frac{d}{2}$.
Answer model: 3 to find the electric field, 2 to get from that the voltage difference.

3. *5 points*

Three charges are situated at the corners of a square (side a), as indicated in the figure on the right. How much work does it take to bring in another charge $+q$ from far away, and put it in the fourth corner?



Answer: problem 2.31 a):

$$V = \frac{1}{4\pi\epsilon_0}\sum\frac{q_i}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\left\{\frac{-q}{a}+\frac{q}{\sqrt{2}a}+\frac{-q}{a}\right\} = \frac{q}{4\pi\epsilon_0 a}(-2+\frac{1}{\sqrt{2}}).$$

Thefore,

$$W_4 = qV = \frac{q^2}{4\pi\epsilon_0 a}(-2+\frac{1}{\sqrt{2}})$$

Answer model: 3 points for calculating the potential correctly, 2 points for relating potential to work done correctly.

4. *2 points*
We observe the electric potential generated by the four-charge arrangement that was assembled in the previous question. At a distance $r >> a$, the potential is proportional to
A. r^{-1}
B. r^{-2}
C. r^{-3}
D. r^{-4}

Answer: a quadrupole potential drops as r^{-3}.
Answer model: all or nothing

5. *5 points*
Find the electric field and the potential inside and outside of a spherical shell of radius R that carries a uniform surface charge. Set the reference point at infinity.
Answer: Example 2.7: From Gauss's law, the field outside is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}},$$

where q is the total charge on the sphere. The field inside is zero. To calculate the potential, we have to integrate the electric field. First for $r > R$:

$$V(r) = -\int_{\infty}^{r}\mathbf{E}\cdot d\mathbf{l} = \frac{1}{4\pi\epsilon_0}\frac{q}{r\prime}\bigg|_{\infty}^{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}.$$

To find the potential in the sphere, we need to integrate first from infinity to the sphere edge, then the inside part, to find

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{R}.$$

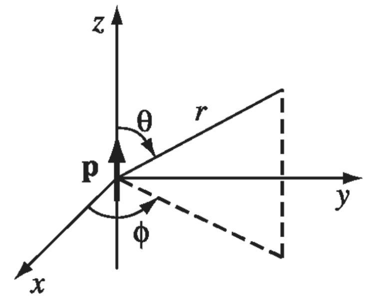
Answer model: 1 point for the electric field, 4 points for the correct potential.

6. *5 points*

The potential for a perfect dipole, located at the origin, pointing in the positive z-direction, is

$$V_{dip}(r,\theta) = \frac{\hat{\mathbf{r}} \cdot \mathbf{p}}{4\pi\epsilon_0 r^2}.$$

Derive the expression for the electric field of this perfect dipole.



Using the negative gradient of V in spherical coordinates, we find

$$\mathbf{E}_{dip}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\theta}).$$

Answer model: 1 to find the explicit θ dependence of the potential, then 4 to correctly take the gradient in spherical coordinates.

7. *5 points*
A long straight wire, carrying uniform charge density λ, is surrounded by rubber insulation (with dielectric constant ϵ_r) out to a radius a. Find the electric field inside and outside the rubber insulation.

Answer: This is like example 4.4, where we found the electric displacement $\mathbf{D} = \frac{\lambda}{2\pi s}\hat{\mathbf{s}}$, *and from that the electric field outside the insulation to be* $\mathbf{E} = \frac{1}{\epsilon_0}\mathbf{D} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{\mathbf{s}}$. *Inside the insulation the field is reduced by the polarization of the dielectric material, and we have* $\mathbf{E} = \frac{\lambda}{2\pi\epsilon s}\hat{\mathbf{s}}$
Answer model: 2 to get the right displacement, 3 to find the right electric fields.

The End